Mathematical Girls - Fermat's Last Theorem

C07

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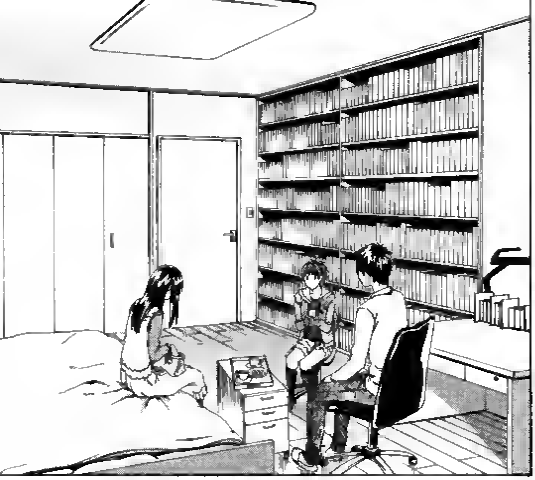
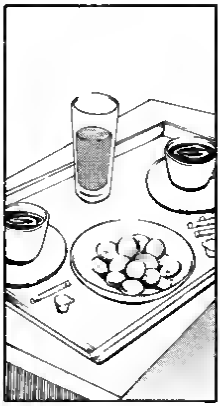
Why?
...Just why...
第 7 話

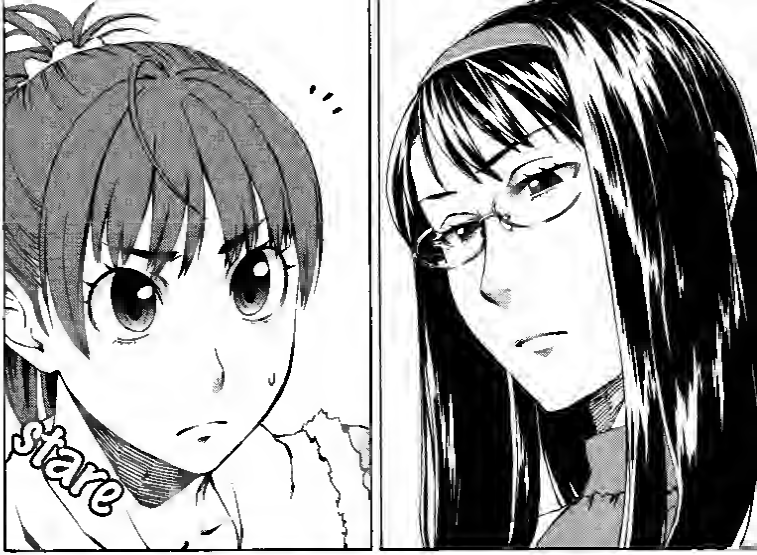
WHY IS THERE ANOTHER WOMAN AT ONII-CHAN'S HOUSE —?!
sparkle
tap
tap
tap
YOU CAME BACK AT A GOOD TIME!
I WILL GO GET THE TEA READY RIGHT AWAY, SO GO WAIT IN THE ROOM
OH YURI-CHAN TOO.

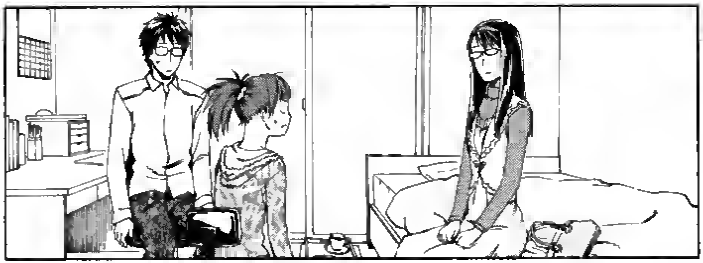


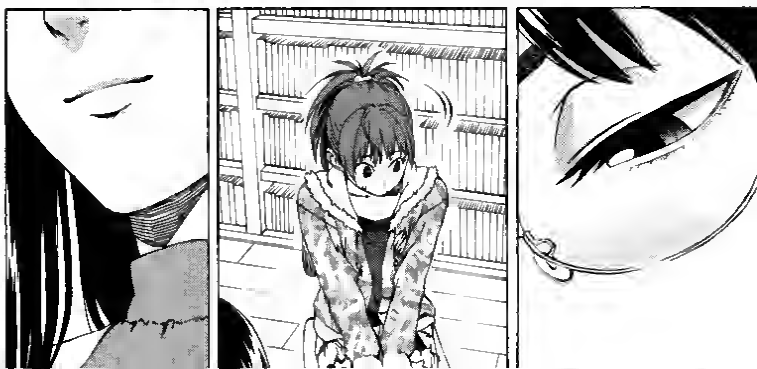


SO SHE CAME ALL THE WAY HERE ON THE TRAIN TO DELIVER IT .
UH, THANK YOU!
Here
YOUR COPY OF THE CAREER SURVEY.
IT WAS IN MY BAG.

.....
AND THIS IS MY CLASSMATE MIRUKA-SAN
bow
UMM,
THIS IS MY COUSIN YURI, A SECOND YEAR MIDDLE SCHOOL STUDENT.

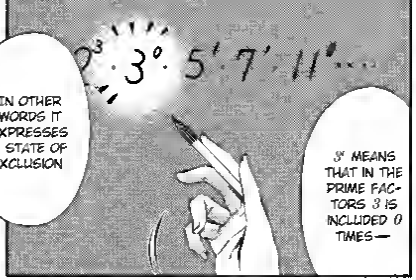

Stare

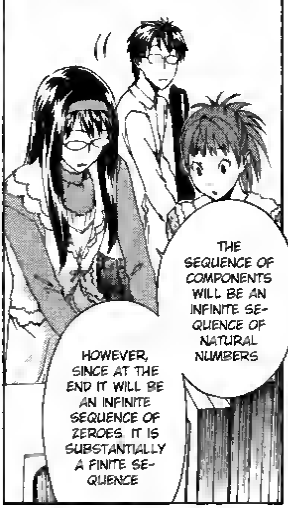




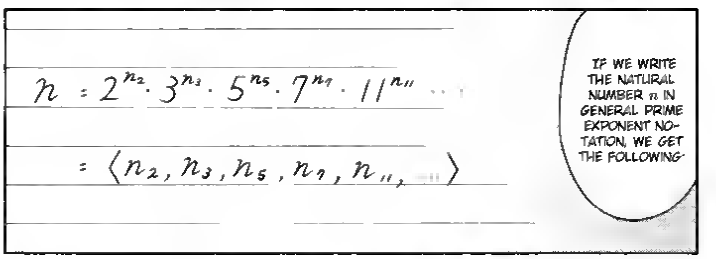
YOU AND YURI RESEMBLE EACH OTHER
OH, IS THAT SO...
OH YEAH, I WAS TEACHING MATHS TO YURI A WHILE EARLIER.
WE WILL BE RESUMING THAT NOW
Hm.
"WHEN M IS THE GREATEST COMMON DIVISOR OF a AND b, AND L IS THEIR LOWEST COMMON MULTIPLE, EXPRESS $a \times b$ IN TERMS OF M AND L."
THAT'S THE PROBLEM WE WERE DOING
M × L

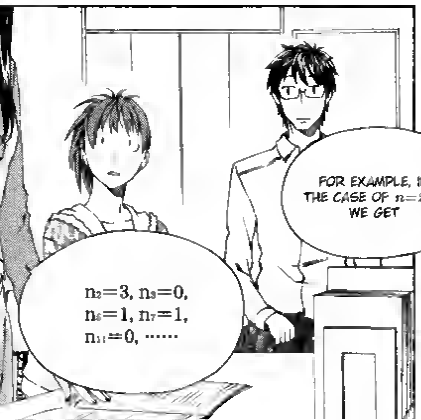
WELL THEN, I SHALL BE TALKING ABOUT PRIME NUMBER EXPONENT NOTATION
WE DECOMPOSE THE NATURAL NUMBERS INTO THEIR PRIME FACTORS AND PAY ATTENTION TO THE EXPONENTS OF THE PRIME FACTORS
FOR EXAMPLE, WE PERFORM THE PRIME FACTOR DECOMPOSITION OF $n=280$ LIKE SO
$280 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7$
We decompose 280 into its prime factors.
$= 2^3 \cdot 3^0 \cdot 5^1 \cdot 7^1 \cdot 11^0 \cdots$
Pay attention to the exponents of the prime factors
$= \langle 3, 0, 1, 1, 0, \cdots \rangle$
Collect together just these exponents
LET'S CALL THIS $\langle 3,0,1,1,0,\cdots \rangle$ FORM OF NOTATION "PRIME EXPONENT NOTATION."
AND CALL EACH ONE OF $3,0,1,1,0,\ldots$ A "COMPONENT".

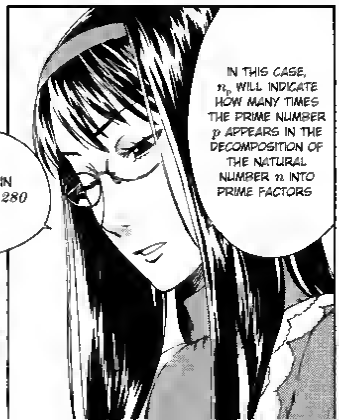

$2^3 \cdot 3^0 \cdot 5^1 \cdot 7^1 \cdot 11^1 \cdots$
IN OTHER WORDS IT EXPRESSES A STATE OF EXCLUSION
3^0 MEANS THAT IN THE PRIME FACTORS 3 IS INCLUDED 0 TIMES—


THE SEQUENCE OF COMPONENTS WILL BE AN INFINITE SEQUENCE OF NATURAL NUMBERS
HOWEVER, SINCE AT THE END IT WILL BE AN INFINITE SEQUENCE OF ZEROES IT IS SUBSTANTIALLY A FINITE SEQUENCE

SINCE 3^0 EQUALS 1, IT IS OKAY TO THINK THAT WE ARE JUST MULTIPLYING BY ONE


$n = 2^{n_2} \cdot 3^{n_3} \cdot 5^{n_5} \cdot 7^{n_7} \cdot 11^{n_{11}} \cdots$
$= \langle n_2, n_3, n_5, n_7, n_{11}, \cdots \rangle$
IF WE WRITE THE NATURAL NUMBER n IN GENERAL PRIME EXPONENT NOTATION, WE GET THE FOLLOWING


FOR EXAMPLE, IN THE CASE OF $n=280$ WE GET
$n_2=3$, $n_3=0$, $n_5=1$, $n_7=1$, $n_{11}=0$, ……


IN THIS CASE, n_p WILL INDICATE HOW MANY TIMES THE PRIME NUMBER p APPEARS IN THE DECOMPOSITION OF THE NATURAL NUMBER n INTO PRIME FACTORS

BECAUSE OF THE UNIQUENESS OF THE PRIME FACTOR DECOMPOSITION, THESE PRIME EXPONENTIAL FORMS CORRESPOND ONE-TO-ONE TO THE NATURAL NUMBERS
WHICH MEANS NO MATTER WHAT A NATURAL NUMBER WAS, IT CAN BE EXPRESSED IN PRIME EXPONENT NOTATION.
AND CONVERSELY, FOR ANYTHING IN PRIME EXPONENT NOTATION, THERE EXISTS A NATURAL NUMBER THAT CORRESPONDS TO IT.

NOW THEN, LET'S GIVE YURI A PROBLEM
WHAT NATURAL NUMBER DOES THE FOLLOWING PRIME EXPONENTIAL FORM EXPRESS?
⟨1, 0, 0, 0, 0, ⋯⟩

2...
Cha
I THINK.

⟨1,0,0,0,0,…⟩ = 2¹·3⁰·5⁰·7⁰·11⁰…
= 2
RIGHT, THIS EQUALS 2.
fu fu fu
YURI FEELS SOMEWHAT DIFFERENT FROM USUAL.
nod
⟨0,1,0,0,0,…⟩ = 2⁰·3¹·5⁰·7⁰·11⁰…
= 3
YEAH.
THAT'S RIGHT
IS IT 3?
NOW THE NEXT PROBLEM HOW ABOUT THIS?
⟨0,1,0,0,0,…⟩

I DON'T KNOW.
NO.
THE SPEED OF YOUR ANSWER IS PROOF THAT YOU DIDN'T THINK.

DO YOU KNOW THIS ONE?
〈1, 0, 2, 0, 0, …〉

PERSEVERE, YURI!

mutter
BUT I DON'T REALLY KNOW.

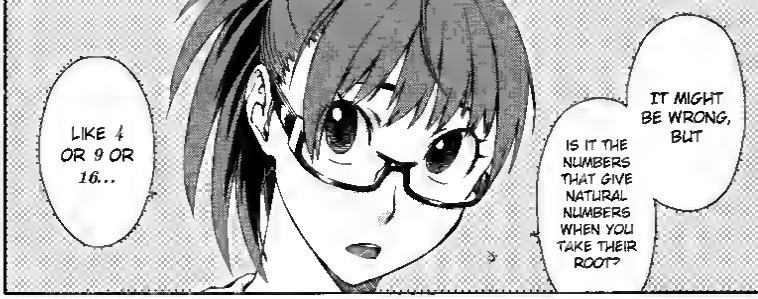
"IF YOU ARE GOING TO SAY SOMETHING WRONG ANYWAY, YOU MIGHT AS WELL SAY THAT YOU DON'T KNOW,"
THAT'S WHAT YOU'RE THINKING, ISN'T IT?
gulp
BECAUSE IT'S SCARY,
YOU ARE JUST AFRAID OF SAYING SOMETHING WRONG.
THUD
YURI CAN ANSWER.
COWARD.

IT'S 27!
WRONG.
THAT STEP AT THE END IS NOT ADDITION.
AH.
I SEE, YOU MULTIPLY THEM.
IT'S 50.
YEAH, NOW THAT'S RIGHT
⟨1,0,2,0,0⋯⟩ = 2¹·3⁰·5²·7⁰·11⁰⋯
= 2·25
= 50
MIRUKA-SAN, YURI GETS IT.
THAT PRIME EXPONENT NOTATION.

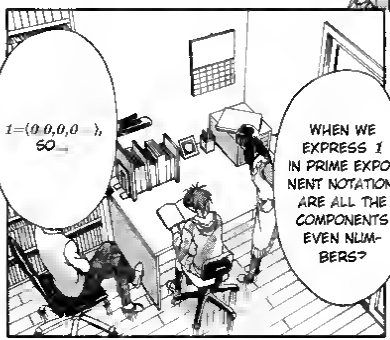
0?
WRONG.
HOW DID YOU CALCULATE IT?
⟨0, 0, 0, 0, 0, ⋯⟩
I DON'T KNOW.
YEAH NOW THEN, THIS ONE?
glare
YURI.
learn learn learn learn
HOW DID YOU CALCULATE IT?
I'M SAYING, SINCE THEY ARE ALL—
SINCE THEY ARE ALL 0,
IT'S 0.
AH!
$2^0, 3^0, 5^0, 7^0, 11^0 \cdots$ THEREFORE THE ANSWER IS 1!
$\langle 0,0,0,0,0, \cdots \rangle = 2^0 \cdot 3^0 \cdot 5^0 \cdot 7^0 \cdot 11^0 \cdots$
$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot \cdots$
$= 1$

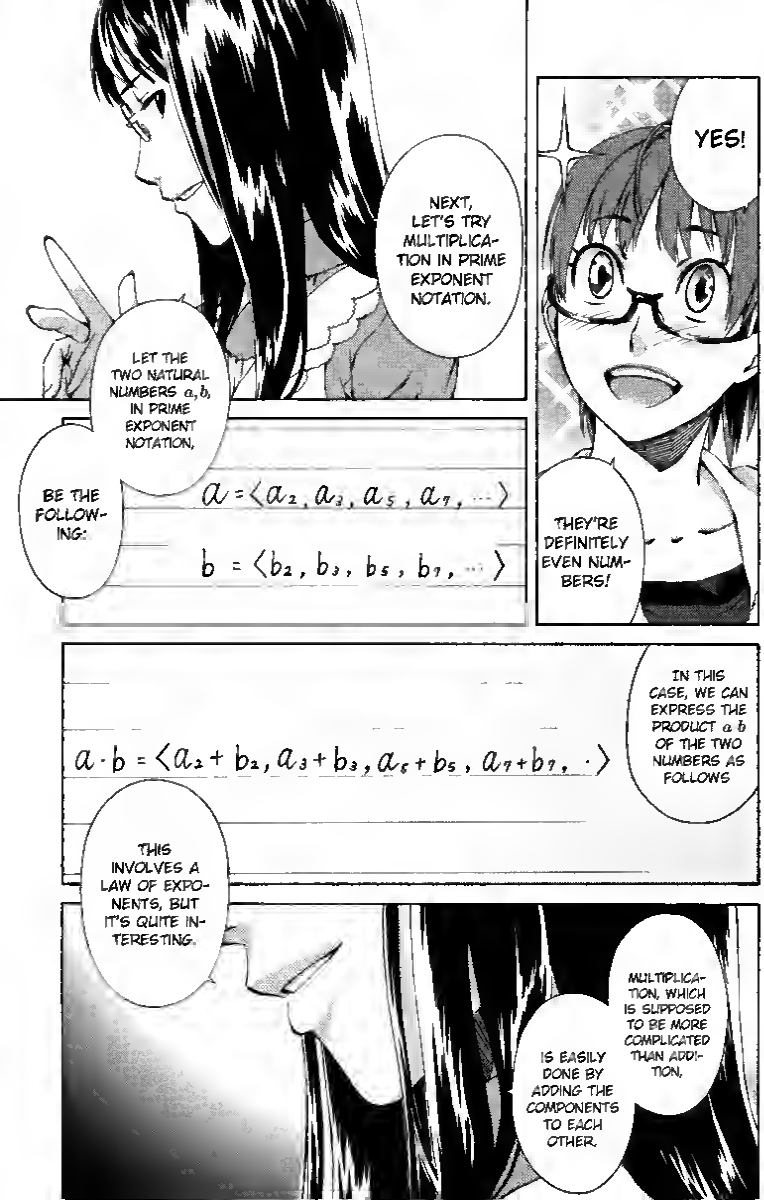
YOU DID REALLY WELL ANSWERING THAT, YURI!

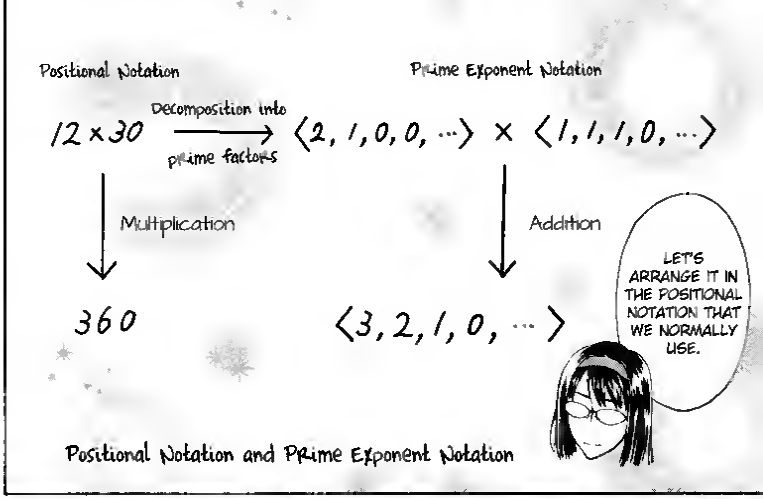
FROM NOW ON THE TEMPO WILL ONLY GO UP.
clack
clack
clack
...A PRIME NUMBER?
AND THE REST ARE 0. WHAT DO WE CALL THIS NUMBER n?
IN AN EXPRESSION $\langle n_2, n_3, n_5, n_7, n_{11}, \rangle$ IN PRIME EXPONENT NOTATION, ONLY ONE COMPONENT EQUALS 1,
indeed
I DON'T KNO...
NO, I WILL THINK
ALRIGHT. NOW THEN, WHAT DO WE CALL A NUMBER WHERE ALL THE COMPONENTS ARE EVEN NUMBERS?
CERTAINLY,
YURI HAS TIMES WHEN SHE INSTANTLY ANSWERS "I DON'T KNOW."
hm-hm
YOU'RE AMAZING, MIRUKA-SAN. YOU'RE GAINING CONTROL OF YURI



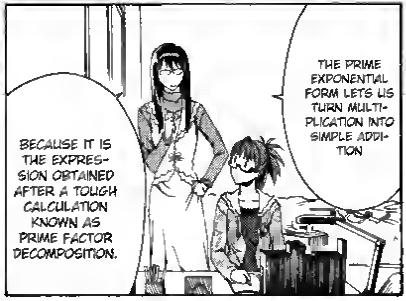
SQUARE NUMBERS




NEXT, LET'S TRY MULTIPLICATION IN PRIME EXPONENT NOTATION.
LET THE TWO NATURAL NUMBERS a, b, IN PRIME EXPONENT NOTATION,
BE THE FOLLOWING:
$a = \langle a_2, a_3, a_5, a_7, \cdots \rangle$
$b = \langle b_2, b_3, b_5, b_7, \cdots \rangle$
YES!
THEY'RE DEFINITELY EVEN NUMBERS!
IN THIS CASE, WE CAN EXPRESS THE PRODUCT $a \cdot b$ OF THE TWO NUMBERS AS FOLLOWS
$a \cdot b = \langle a_2 + b_2, a_3 + b_3, a_5 + b_5, a_7 + b_7, \cdots \rangle$
THIS INVOLVES A LAW OF EXPONENTS, BUT IT'S QUITE INTERESTING.
MULTIPLICATION, WHICH IS SUPPOSED TO BE MORE COMPLICATED THAN ADDITION,
IS EASILY DONE BY ADDING THE COMPONENTS TO EACH OTHER.

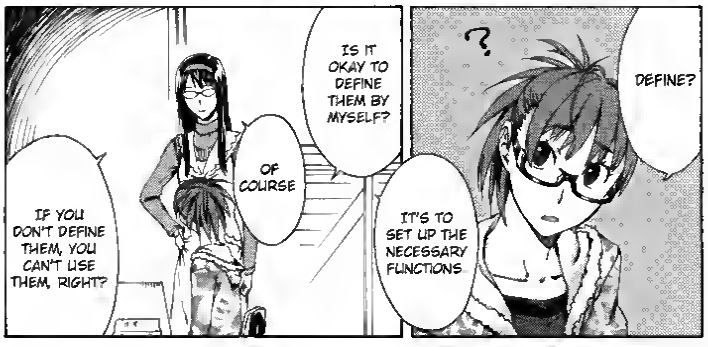

Positional Notation
Prime Exponent Notation
12×30
Decomposition into prime factors
⟨2, 1, 0, 0, ⋯⟩ × ⟨1, 1, 1, 0, ⋯⟩
Multiplication
Addition
360
⟨3, 2, 1, 0, ⋯⟩
LET'S ARRANGE IT IN THE POSITIONAL NOTATION THAT WE NORMALLY USE.

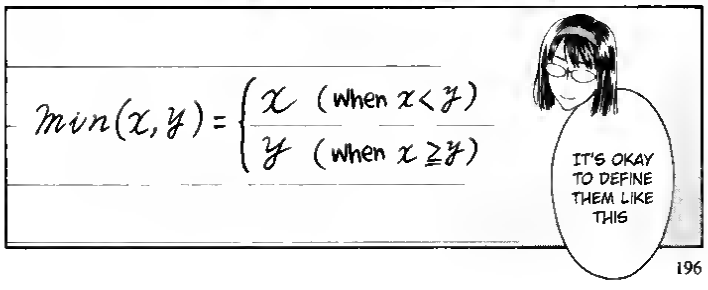
Positional Notation and Prime Exponent Notation


BECAUSE IT IS THE EXPRESSION OBTAINED AFTER A TOUGH CALCULATION KNOWN AS PRIME FACTOR DECOMPOSITION.
THE PRIME EXPONENTIAL FORM LETS US TURN MULTIPLICATION INTO SIMPLE ADDITION

$a = \langle a_2, a_3, a_5, a_7, \cdots \rangle$
$b = \langle b_2, b_3, b_5, b_7, \cdots \rangle$
WELL THEN, NOW FOR THE GREATEST COMMON DIVISOR
TRY TO THINK ABOUT IT, YURI.
CAN WE EXPRESS THE GREATEST COMMON DIVISOR OF THE TWO NATURAL NUMBERS a AND b USING PRIME EXPONENT NOTATION?
.....
MIRUKA-SAN, IS IT A PROBLEM IF I DON'T WRITE IN MATHEMATICAL EQUATIONS?
YURI CAN'T WRITE THIS WITH THE EQUATIONS SHE KNOWS.
TRY TO SAY WHAT YOU WANT TO EXPRESS IN JAPANESE.
I WANT TO WRITE "THE ONE OF TWO NUMBERS THAT'S LESS THAN THE OTHER", BUT...
THE ONE LESS THAN THE OTHER?
OR THE ONE THAT'S NOT BIGGER?
EH...
YES! THE ONE THAT'S NOT BIGGER!

min(x,y) = (out of x and y, the number that's not bigger.)
IF THERE'S SOMETHING YOU WANT TO WRITE, YOU SHOULD JUST DEFINE A NEW FUNCTION.
FOR EXAMPLE THAT'S HOW WE DEFINE THE FUNCTION CALLED $\min(x,y)$.


?
DEFINE?
IT'S TO SET UP THE NECESSARY FUNCTIONS
IS IT OKAY TO DEFINE THEM BY MYSELF?
OF COURSE
IF YOU DON'T DEFINE THEM, YOU CAN'T USE THEM, RIGHT?


$min(x,y) = \begin{cases} x & (\text{when } x < y) \\ y & (\text{when } x \geq y) \end{cases}$
IT'S OKAY TO DEFINE THEM LIKE THIS

(The greatest common divisor of a and b) =
$\langle min(a_1,b_2), min(a_2,b_3), min(a_5,b_5), min(a_7,b_7), \cdots \rangle$
THE GREATEST COMMON DIVISOR CAN BE EXPRESSED USING $\min(x,y)$
THAT'S GOOD, YURI.
...I SEE.
THEN, TOGETHER WITH THE VECTORS*,
YOU JUST HAVE TO DEFINE IT YOURSELF
LET'S HEAD OUT TO AN INFINITE-DIMENSIONAL SPACE.

TL Note: Miruka is using the actual english pronunciation of the word "vector" instead of the derived Japanese equivalent "bekutoru". Originally there was a note that had the Japanese equivalent so that Japanese readers can understand what Miruka is saying.

SEEKING THE GREATEST COMMON DIVISOR MEANS
SEEKING $\min(a_p, b_p)$ FOR EVERY COMPONENT IN THE PRIME EXPONENTIAL FORMS.
HAVING "a AND b BE COPRIME" WILL CORRESPOND TO "FOR ALL PRIME NUMBERS p, $\min(a_p, b_p)=0$."
a and b are "coprime"
⇕
for all prime numbers p, min(a_p, b_p)=0
IN OTHER WORDS THIS MEANS THAT FOR ALL PRIME NUMBERS p, ONE OF a_p AND b_p MUST BE EQUAL TO 0.
WE CAN ALSO SAY THAT THE TWO VECTORS WILL NEVER CAST SHADOWS ON THE SAME COORDINATE AXIS.
EVERY AXIS CORRESPONDS TO A PRIME NUMBER, AND THE COORDINATES FOR EACH AXIS WILL BE THE COMPONENTS n_2, n_3, n_5, n_7,...
THE PRIME NUMBER EXPONENTIAL FORMS ARE CONSIDERED INFINITE DIMENSIONAL VECTORS.
SINCE IT'S AN INFINITE-DIMENSIONAL SPACE, THERE ARE AN INFINITE NUMBER OF COORDINATE AXES.
AND DECOMPOSING A NATURAL NUMBER INTO ITS PRIME FACTORS
CORRESPONDS TO DISCOVERING THE SHADOW CAST BY THIS POINT ON THE COORDINATE AXES.
A NATURAL NUMBER WILL CORRESPOND TO ONE POINT IN THAT INFINITE-DIMENSIONAL SPACE.
THEN WHAT COULD THE STATE OF HAVING TWO NATURAL NUMBERS BE "COPRIME" CORRESPOND TO GEOMETRICALLY?
1
AND 1 IS $(0, 0, 0, 0, \ldots)$ IF EXPRESSED IN PRIME EXPONENT NOTATION.
IF TWO NUMBERS ARE "COPRIME" THEN THEIR GREATEST COMMON DIVISOR WILL EQUAL 1.

IN SHORT, THIS IS WHAT'S CALLED HAVING TWO VECTORS BE ORTHOGONAL.
BASED ON THIS, THERE ARE SCHOLARS WHO WRITE "a AND b ARE COPRIME"
AS $a \perp b$.
a and b are "coprime"
⇕
$a \perp b$
SINCE ⊥ EVOKES THE IMAGE OF ORTHOGONALITY
"COPRIME" IS A NUMBER-THEORETIC EXPRESSION,
WHILE BEING "ORTHOGONAL" IS A GEOMETRIC EXPRESSION

GEOMETRY PROVIDES US WITH MANY EXPRESSIONS.

HEY,
SHE'S THE ONE WHO GAVE THAT "FIND-THE-ODD-ONE-OUT-QUIZ", ISN'T SHE?
YEAH. HOW DID YOU KNOW?
IT'S HER WRITING!
THE HAND-WRITING!!
MIRUKA-SAMA SURE IS AMAZING
MIRUKA-SAMA?
HAVING LONG, BEAUTIFUL BLACK HAIR IS JUST UNFORGIVABLE
sulk
A-H.
MAYBE I SHOULD LET MY HAIR GROW LONGER
sulk

Hfuu
YURI MIGHT BE ADMIRING HER...
HOW IS SHE ABLE TO SAY ALL THAT WITH SUCH TENACITY.
IS TETRA-CHAN SUCH AN AMAZING PERSON AS WELL...
Uwah, it's even smaller this time
WAS TALKING ABOUT TETRA-CHAN ON HER WAY OUT.
COME TO THINK OF IT, MIRUKA-SAMA
YOU DID CALL HER OUT IN THE HOSPITAL, DIDN'T YOU? WHAT EXACTLY DID YOU TELL HER AT THAT TIME?
SPEAKING OF TETRA-CHAN,
EEH...
Jump-up

Yuri is a fourth degree relative.

—What is it that this definition seeks means?

第2巻へつづく

Yuuki Hiroshi's Extra Lectures 7

Prime Factor Decomposition and Prime Number Exponent Notation

The prime factor decomposition of natural numbers means dismantling a natural number into a product of prime numbers.

For example, since the natural number 280, as 2×2×2×5×7, can be dismantled into a product of the prime numbers 2, 5 and 7, so 2×2×2×5×7 is called the prime factor decomposition of 280. In this case, 2, 5 and 7 are the prime factors of 280.

In the prime factor decomposition of 2×2×2×5×7, we have the prime number 2 three times, 3 zero times, 5 once, 7 once,... multiplied together. We express that as $2^3 \times 3^0 \times 5^1 \times 7^1 \times \ldots$

The 2,3,5,7,11,... are prime numbers and the small numbers put on the upper-right side of these prime numbers are called "exponents". These exponents, when put in order, (3,0,1,1,0,) will be 280 in prime exponent notation.

Prime number exponent notation is something that we don't learn at school, but it's a form of writing that nicely expresses the prime factor decomposition.

After being scolded by Miruka-san, Yuri started to think correctly about prime number exponent notation.

If we understand each thing properly as we go on, maths, too, will be nothing to fear.

Postscript

This work is a manga version that Kasuga Shun-sensei has reorganized based on "Mathematical Girls: Fermat's Last Theorem", the second volume of the "Mathematical Girls" series. Seeing Miruka-san's figure as a teacher, or Tetra-chan's Alice-style dress was nice, but seeing Yuri closing her eyes on the back of the bike was a big hit for me.

Everyone please, together with the mathematical girls, enjoy the world of mathematics!

Enjoy!

Yuki Hiroshi

"Mathematical Girls" series homepage

http://www.hyuki.com/girl/

Postscript

Hello, I'm Kasuga Shun, the mangaka drawing this manga adaptation.

Come to think of it, it was a few years ago.

I was working on a manga called "Ane to Muchi".

At that time, the manga adaptation of the previous work "Mathematical Girls" was being published in the same magazine; I never imagined that I would be the one to draw its sequel.

And since I am clueless about even basic mathematics, I decided the plot and the storyline while consulting my assistant and, needless to say, the maths parts were written under the complete supervision of Yuki-sensei.

"Mathematical Girls: Fermat's Last Theorem" was made by the combined efforts of so many people. Including the readers, thank you all very much.

See you again in volume 2.

Also, I'm in Tetra-chan's fan club.

2011.04 Kasuga Shun